Analysis of Transient Electric Field and Charge Density of Converter Transformer under Polarity Reversal Voltage

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***Abstract*—In this paper a method of computing the boundary charge density and the normal field strength on the Dirichlet boundaries under the PR (polarity reversal) voltage is presented. The method adopts C-N (Crank-Nicolson) method to solve the transient equation and uses directly the node charges as variable to obtain the boundary node charge densities, which could be adopted to compute the normal electric field strength. The method is verified by a two layer coaxial model and applied to analyze the transient electric fields and boundary charge densities of a converter transformer under PR voltage successfully.**

I. INTRODUCTION

Converter transformer is one of key equipments in HVDC substation. The electric fields of converter transformer under PR (polarity reversal) voltage, which is much more complex than AC or DC static electric fields, have been studied by many researchers. When DC voltage applied on the valve side windings of the transformer, charges would accumulate on the interfaces of insulation materials and seriously influence the electric field distribution when PR happens. Although many papers on the PR's electric field analysis of converter transformer have published [1]-[2], the quantitatively numerical studies on charges or charge densities under PR have not done.

In Ref. [3], a charge-electric potential formulation is used to analyze transient nonlinear electric field, in which the dynamic node charges and potentials can be got simultaneously. However we usually get the charge density from the actual measurement voltage, so the charges should be converted to charge densities, which are easy to compare with the corresponding ones calculated from measurement.

In this paper, a method of obtaining the volumetric and surface charge densities is presented, and the PR's electric field represented by a scalar electric potential and charge densities is solved by the C-N method. The surface charge densities on the Dirichlet boundaries are adopted to get the normal electric field with higher accuracy, which are very useful in insulation design of converter transformer. The accuracy of the method is tested by a simple linear model, and transient electric fields, surface charge densities of a converter transformer are calculated.

II. FORMULATION

A. Charge Density –Potential Formulation

The quasi-static electric field can be governed by the following equations：

$$\boldsymbol{E} = -\nabla\varphi \tag{1}$$

$$\frac{\partial\rho}{\partial t} + \nabla\cdot\boldsymbol{J} = 0 \tag{2}$$

$$\rho - \nabla\cdot\boldsymbol{D} = 0 \tag{3}$$

The surface charge exists on the interface of two media. The surface charge satisfies:

$$\frac{\partial\rho_s}{\partial t} + \left[\gamma_1\frac{\partial\varphi_1}{\partial n} - \gamma_2\frac{\partial\varphi_2}{\partial n}\right] = 0 \tag{4.1}$$

$$\rho_s - \left(\varepsilon_1\frac{\partial\varphi_1}{\partial n} - \varepsilon_2\frac{\partial\varphi_1}{\partial n}\right) = 0\,. \tag{4.2}$$

Applying Galerkin method, we could obtain the weak form equations:

$$\int_\Omega N_i\frac{\partial\rho_h}{\partial t}d\Omega + \int_{\Gamma_{int}} N_i\frac{\partial\rho_{s,h}}{\partial t}d\Gamma + \int_\Omega\nabla N_i\cdot\gamma\nabla\varphi_h d\Omega = 0 \tag{5.1}$$

$$\int_\Omega N_i\rho_h d\Omega + \int_{\Gamma_{int}} N_i\rho_{s,h}d\Gamma - \int_\Omega\nabla N_i\cdot\varepsilon\nabla\varphi_h d\Omega = 0 \tag{5.2}$$

In Ref. [3], the charge density integration terms in the above equation are treated as node charges. We can directly use the volume charge density and surface charge density as variables in (5), which will get the following equations:

$$[\boldsymbol{P}]\frac{\partial\{\boldsymbol{\rho}\}}{\partial t} + [\boldsymbol{K}_\gamma]\{\boldsymbol{\varphi}\} = 0 \tag{6.1}$$

$$[\boldsymbol{P}]\{\boldsymbol{\rho}\} - [\boldsymbol{K}_\varepsilon]\{\boldsymbol{\varphi}\} = 0 \tag{6.2}$$

Where $\boldsymbol{K}_\gamma$, $\boldsymbol{K}_\varepsilon$ are the dielectric matrix and conductivity matrix, respectively. $\boldsymbol{P}$ contains two kinds' coefficients, which are called volumetric coefficient P_v and surface coefficient P_s in this paper. For a two dimensional plane problems, P_v and P_s can be computed by [4]

$$P^e_{vij} = \int_{\Omega^e} N^e_i N^e_j d\Omega = \frac{\left(1+\delta_{ij}\right)}{12}\Delta^e \tag{7.1}$$

$$P^e_{sij} = \int_{\Gamma^e} N^e_i N^e_j d\Gamma = \frac{\left(1+\delta_{ij}\right)}{6}l^e \tag{7.2}$$

There are only surface charges on the surface. We could treat all the volumetric charge densities as the homogeneous Dirichlet boundary conditions and impose them as we deal with the potential boundary conditions in FEM. For example, if i is a surface node, all the volumetric coefficients P_{vij} related with node i are set to zeros and P_{vii}=1, the coefficient P_{vij} is then replaced by P_{sij} due to ρ_{vi}=0. Therefore, the steps of computing $\boldsymbol{P}$ are:

a. Compute volumetric coefficient P_{vij} and assembly them into $\boldsymbol{P}$.

b. Set all the volumetric coefficients related to the surface node to zeros.

c. Compute surface coefficient P_{sij} and assembly them into P.

The $\boldsymbol{P}$ in (7) could be repeatedly used at each time step due to the independence of material property.

Writting (7) into a more compact matrix form:

$$\begin{bmatrix} \boldsymbol{P} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} \begin{Bmatrix} \dfrac{\partial \boldsymbol{\rho}}{\partial t} \\ \dfrac{\partial \boldsymbol{\varphi}}{\partial t} \end{Bmatrix} + \begin{bmatrix} 0 & \boldsymbol{K}_{\gamma} \\ \boldsymbol{P} & -\boldsymbol{K}_{\varepsilon} \end{bmatrix} \begin{Bmatrix} \boldsymbol{\rho} \\ \boldsymbol{\varphi} \end{Bmatrix} = 0 \qquad (8)$$

Equation (8) is an initial value problem with the specified boundary and initial conditions and can be solved by C-N method [5].

B. Computation of Boundary Charge Density and Normal Electric Field Strenth

The normal electric field intensities on the Dirichlet boundaries at the *k*-th time step can be obtained by

$$E_{n\,i}^{k} = \rho_{i}^{k} / \varepsilon_{i}\,, \qquad (9)$$

where ρ_i^k and ε_i are the surface charge density at *k*-th time step and the permittivity at the normal direction of the node *i* respectively.

III. Verification

The accuracy of the proposed method can be shown by a coaxial model consisted of two layers dielectrics under DC voltage in Fig.1.

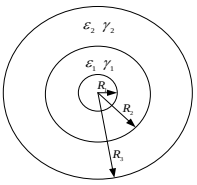

Fig. 1 Dual layer dielectrics of coaxial model, R_1=1m, R_2=3m, R_3=7m.ε_1 =2ε_0, γ_1 =10^{-15}S/m, ε_2 =ε_0, and γ_2 =10^{-13}S/m.

The amplitude of a ramp voltages applied on inner conductor (R_1) is 1000V and outer conductor (R_3) is grounded. Only one fourth areas are analyzed due to the axial symmetry. The area is divided into triangular meshes. Two different meshes are used in our analysis, (a) Coarse mesh: 340 triangles, 196 nodes, (b) Fine mesh: 8554 triangles, 4401 nodes.

The analytical solution of only the surface density and the maximum and minimum relative error of the proposed method are displayed in Fig. 2 for simplicity, the detail will be given in the full paper.

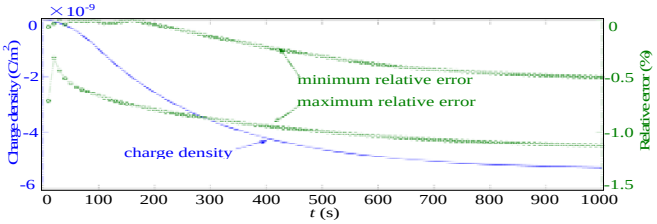

Fig. 2 Curves of charge density and its relative errors with coarse meshes

The curves show that when the system tends to be steady state, the relative errors would reach its maximum. So we give the relative errors of the normal electric field intensity on R_1, R_3 and the surface charge density on R_2 under steady state. We also give the results of common FEM post-process method, which the node electric strength is obtained by dividing the summation of the related elements' electric fields by the elements' total number, in Tab. I.

TABLE I
THE RELATIVE ERROR OF THE TWO METHODS

		E_r at R_1 (AV)		Relative error of E_r at R_3		Relative error of ρ_s at R_2	
		proposed	traditional	proposed	traditional	proposed	traditional
Coarse mesh	r_{max}	4.10	-16.91	0.747	4.50	1.149	10.169
	r_{min}	-0.355	-17.98	-0.145	3.9	-0.707	8.67
Fine mesh	r_{max}	0.804	-4.0	1.435	0.729	-0.240	1.717
	r_{min}	-0.858	-4.87	-0.0863	0.605	0.305	1.431

From Table. I we can clearly see that the precision of the common FEM method with coarse meshes is very low and can be improved by refining the meshes, which need much more computational cost. The proposed method gets much higher accuracy because it does not need differentiation as the common FEM post-process.

Though the accuracy of the presented method is only testified by a simply linear model, which has no volume charges, the method to solve (6.2) could be regarded as an inverse operation of calculating the source vector from the kwon charge density distribution [6], which is commonly used in solving Poisson's equation. So the accuracy of the volumetric densities would be guaranteed according to the potential distributions at each time step.

IV. Numerical Application

The transient electric fields and charge densities of a converter transformer under the PR voltage are computed by the presented method. The transient surface node charge densities are used to calculate the normal electric fields on the electrostatic ring and the bushing barrier. The detail will be given in the full paper.

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